

## Problem Solving and Reasoning Questions

### Guidance

**These problem solving and reasoning questions have been taken from our therapies. They have been grouped according to subject for your convenience. However, when sharing them with pupils it is important to mix the problems so that pupils have to decipher which area of maths they will have to use to solve the problems.**

### Number and Place Value

Can you make a number that has six tens, five thousands and is between 2,000,000 and 3,000,000?

If you add 1,000 to 14,357,999 what number would you get?

Which number is bigger 4056 or 4506? Explain why.

How many steps of 10 are there from the number 13950 to the number 14030?

Tell me a number that lies between 3.12 and 3.13. Which number is it closer to? How do you know?

Kyle runs the 200m in 25.7 seconds. Harry runs 12 seconds slower than Kyle. What time did Harry run the 200m in?

Stephanie has a plank of wood measuring 2.3m long. She wants to make shelves out of it that are 45cm long each. How many shelves will she be able to cut from her plank of wood? How would you work out what was left if any?

What is the highest number that will round to the nearest ten to give an answer of 160?

The answer was 7300. What rounding question could have been asked? Why?

I think of two numbers. Both are decimals with one decimal place. They both round to 26. What could they be?

When I add my two numbers the total is 52. What could my numbers be?

34.7 and 34.70 have the same value? True or False? Explain your answer.

How can you use  $6 \times 100 = 600$  to work out  $60 \times 10$ ?

Show me a number that, when multiplied by 10, gives an answer greater than 132.

Joe says the answer to  $25 \times 100$  and  $250 \times 10$  is the same. Is he right?

A sheet of paper is 34mm thick. There are 1,000 sheets in a pack. How thick is the pack of paper?

How could you use  $7.4 \times 10 = 74$  to work out  $0.74 \times 100$ ?

Show me the number that, when multiplied by 10, gives me an answer of 82.

Show me the number that, when multiplied by 100, gives me an answer of 82.

Show me the number that, when multiplied by 1000, gives me an answer of 82.

Katie says that the answer to  $9.6 \times 100$  is the same as  $0.96 \times 1000$ . Is she right?

A bead on a string is 1.2cm long. There are 100 on the string. How long is the string?

How can you use  $6 \div 100 = 0.06$  to work out  $60 \div 10$ ?

Show me a number that when divided by 10 gives an answer less than 7.4.

Joe says the answer to  $25 \div 100$  and  $250 \div 10$  is the same. Is he right?

A pile of 1000 sheets of paper measures 14cm. What is the width of one piece of paper?

Explain how you would multiply a decimal by 10 and divide a decimal by 100.

Show me a number that when divided by 100 gives an answer less than 2.5

I divide a number by 10 and then again by 10. My answer is 0.3. What was my starting number?

Amy says the difference between -5 and -19 is 24. Is she correct? How do you know?

The answer is -12 degrees. Make up some calculations that will give this answer.

I measured the temperature outside in the morning. By evening the temperature had fallen by 5 degrees and was below freezing point. What could both of the temperatures have been?

Seb says -8 is bigger than -4. Explain where he has gone wrong.

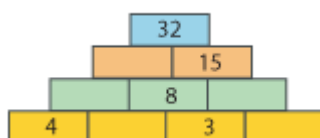
### Number- Addition, Subtraction (mental and written)

I buy six items costing 76p, 89p, 36p, £1.03, 49p and 97p. I give the shop assistant a £10 note and get £3.46 change. I immediately think the change is wrong. Without doing any calculating, explain why you think I am right.

I added the distances I travelled to work for one week. On Monday I travelled 145metres, on Tuesday I travelled 10km, on Wednesday I travelled 25km, on Thursday I travelled 145metres and on Friday I travelled 50km. I entered  $145 + 145 + 10 + 25 + 50$  into my calculator and I got 375 km. That doesn't seem right. What have I done wrong? Is using a calculator the best way to solve this problem? Explain your thinking.

I have 360 ml of juice in a jug. I add another 450 ml. How much juice is there in the jug now?

Solving number puzzles, e.g. Completing number pyramids.



My sister is 108cm tall and I am 132cm. I did a subtraction calculation and worked out that I am 36cm taller than my sister. That doesn't seem right. What do you think I may have done wrong? What is the best way to solve this problem?

Ask pupils to carry out calculations where known facts and place value can be used to add and subtract a pair of numbers mentally, e.g. Using knowledge of  $62 - 38$  to calculate  $6200 - 3800$  or  $6.2 - 3.8$ .

My Christmas credit card bill has just arrived. I have spent £367 on presents, £186.24 on food and £67.82 on decorations. How much have I spent in total?

345 people buy tickets to see a film at the cinema on Friday, 267 watch the film on Saturday and a further 186 watch the film on Sunday. How many people watched the film in total?

Sanjay and Eloise are saving to go on holiday. Sanjay has saved £245.67 and Eloise has saved £342.20. How much have they saved in total?

The Smith family has saved £675 towards their summer holiday. The cost of the holiday is £2019. How much more do they need to save?

At the beginning of a cricket match there were 742 people watching. At tea-time 218 people went home. How many were left?

Two adults and two children go to a cinema. Adult tickets cost £5.85 and children's tickets cost £2.85. How much change will they get from a £20 note?

Max jumped 2.35 metres on his second try at the long jump. This was 68 centimetres longer than on his first try. How far in metres did he jump on his first try?

Would you use column subtraction to work out time intervals? Explain your answer?

I buy some items costing £3.76, 89p, £1.05 and £17.49. How much do I spend in total?

The measurements of my room are 5.45m, 6.75m, 6m and 4.84m. What is the perimeter of my room?

I buy some items costing £14.67. I give the shop assistant a £20 note. What change should I get? How did you work it out?

1 litre of paint covers 11.5 square metres. I paint an area of 5.12 square metres. How much paint do I have left?

## Number- Multiplication and Division

Establish whether pupils are able to connect division facts with finding fractions of numbers and quantities e.g.

$\frac{1}{3}$  of 24 is equivalent to  $24 \div 3$ .

$\frac{4}{5}$  of 35 is equivalent to finding  $35 \div 5$  and multiplying the answer by 4.

Establish whether pupils are able to use factors to help them work out division calculations.

e.g.  $90 \div 6$  by dividing 90 by 3 and dividing the resulting quotient by 2.

The perimeter of an equilateral triangle is 120 cm. How long is each side?

Which numbers less than 100 have exactly three factors?

What number up to 100 has the most factors?

The sum of four even numbers is a multiple of 4. When is this statement true? When is it false?

Can a prime number be a multiple of 4? Why?

Can you give me a number greater than 100 that is divisible by both 3 and 5. How did you do it?

What is  $0.9 \times 4$ ? How did you work this out?

Multiply 7 by 0.6.

Divide 4.8 by 6.

What is  $4 \times 0.7$ ? How do you know?

$0.15 \div 3 =$

What number multiplied by 8 equals 4.8?

What multiplication and division facts do you know, or can you derive, with an answer of 5.6?

What could the missing numbers be?  $\square \times \square = 3.2$

Can you think of several pairs of numbers that would work?

Solve  $2.8 \div \square = 0.4$ .

My pencil is 9.2 cm long. My desk is 6 pencil-lengths wide. How wide is the desk, in centimetres?

Six packets of crisps cost £1.20. How much does each packet cost in £s?

How many different multiplication and division facts can you find, using what you know about 72? What if you started with 7.2? 0.72?

The answer to a multiplication or division calculation is 0.56. What could the calculation be? How many possibilities can you find?

Are any of the following incorrect?

$0.7 \times 0.8 = 5.6$

$$8 \times 0.8 = 6.4$$

$$56 \div 0.7 = 8$$

Explain why and how you know, using words or diagrams.

Establish whether pupils are able to connect division facts with finding fractions of numbers and quantities e.g.

$\frac{1}{3}$  of 240 is equivalent to  $240 \div 3$ .

$\frac{4}{5}$  of 350 is equivalent to finding  $350 \div 5$  and multiplying the answer by 4.

How many £10 notes are in £120, £1200? How many £1 coins, 10p coins, 1p coins?

Tins of dog food at 42p each are put in packs of 10. Six packs are put in a box. How much does one box of dog food cost? 6 boxes? 12 boxes? Etc.

My pencil is 9.2 cm long. My desk is 6 pencil lengths wide. How wide is the desk in centimetres?

Find a number whose double lies between 1.3 and 1.4.

The answer to a multiplication calculation is 0.56. What could the calculation be?

A girl worked out the cost of eight bags of apples at 47p a bag. Her answer was £4.06. Without working out the answer, say whether you think it is right or wrong.

Explain how you would do this multiplication by using factors, e.g.  $5.8 \times 40$

What clues do you look for when deciding if you can do a multiplication mentally? E.g.  $5.8 \times 40$ .

Give an example of how you could use partitioning to multiply a decimal by a two-digit whole number, e.g.  $5.3 \times 23$ .

Discuss the relationship between division and fractions: relate finding  $\frac{1}{3}$  of 24 to  $24 \div 3$

Find fractions of numbers and quantities, e.g. 'What is  $\frac{4}{5}$  of 35?'

I divide a four-digit number by 100. The answer is between 70 and 75. What could the four-digit number be?

Tom says: 'If I divide a four-digit number by 1000 it always has a number after the decimal point.' Is he right? Explain your answer.

The perimeter of an equilateral triangle is 12 cm. How long is each side?'

How do you know if a number is divisible by 6?

Is there a quick way to check if a number is divisible by 25?

The quotient is 5. Make up some questions. How did you go about devising these questions?

$73.6 \div 3.2 = 23$ . Explain how you can use this to devise calculations with the same answer.

Solve missing number problems such as  $4.8 \times \square = 43.2$

611 is the product of two prime numbers. One of the numbers is 13. What is the other one?

Three bars of chocolate cost 90p. How much would six bars cost? And 12 bars?

There is space in the car park for 17 rows of 32 cars. How many cars can park?

How many hours are there in one year?

Show me your method for solving this problem:

'What is the total mass of 235 screws each weighing 6 grams?'

What approximations did you make? Explain how you worked out the answer.

Here is a multiplication calculation with some missing numbers.

x			
30	?	120	
	40	?	

	120
	40
+	
=	768

x = 768

What is the calculation?

Explain how you worked it out.

I buy six books that cost £7.99 each and four CDs that cost £12.99 each. Use approximation to work out the total cost to the nearest pound.

A lawn is 19.5 m long and 4.5 m wide. Is its area greater or less than 100 square metres? Explain how you know.

A car park has 76 rows for parking. There are 52 car spaces in each row. Which of these is the best way to estimate how many cars can park altogether?  $80 \times 60 = 4800$ ,  $80 \times 50 = 4000$ ,  $70 \times 60 = 4200$ . Explain your choice.

Which of these two numbers multiplied together give the product closest to 24? 7.9, 9.2, 2.1, 2.8.

Solve missing number problems

$$4.8 \div \square = 0.96$$

$$1/8 \text{ of } \square = 40$$

The perimeter of a regular octagon is 348cm. How long is each side?

A boy worked out how many 19p stamps you can buy for £5. His answer was 25. Do you think he was right or wrong? Why?

Will the answer to  $75 \div 0.9$  be smaller or larger than 75? How do you know?

317 people are going on a school coach trip. Each coach will hold 28 passengers. How many coaches are needed?

A full box holds 180 pins. How many boxes can be filled from 100 000 pins?

A rope is 12 metres long. How many lengths of 85 cm can I cut from it?

A bus holds 52 people. How many buses are needed for 327 people?

Apples weigh about 190g each. How many apples would you expect to get in a 2 kg bag?

Roughly how many pot plants can I buy with £50 if each plant costs £2.99?

I bought some pencils that cost 15p each. I paid £5.85. How many pencils did I buy?

Three bars of chocolate cost £1.24. How much  would six bars cost?

There is space in the car park for 9 rows of 48 cars. How many cars can park?

Show me your method for solving this problem:

'What is the total weight of 8 apples each weighing 50.4 grams?'

What approximations did you make? Explain how you worked out the answer.

I buy six books that cost £7.79 each and four CDs that cost £12.49 each. Use approximation to work out the total cost to the nearest pound.

A lawn is 19.52 m long and 6 m wide. Is its area greater or less than 100 square metres? Explain how you know. What is its area?

If you multiply me by 3, you will get 24. What number am I?

Multiples of 6 end in 0, 2, 4, 6 or 8. Is this statement true or false?

Multiples of 7 are all even. Is this statement true or false?

Name all the multiples of 7 between 20 and 30.

Name three numbers that are multiples of 6 and multiples of 5.

What is the lowest common multiple of 4 and 6?

Write down the first five numbers that are multiples of 6 and multiples of 8. Describe what you notice about the sequence and predict the next two common multiples.

I need to pay 51p postage using only 12p and 5p stamps. How many of each should I put onto my parcel?

Where in this Carroll diagram should the number 8 go? Write an appropriate number into the bottom left cell.

	multiple of 5	not a multiple of 5
factor of 120		
not a factor of 120		

Find a number between 230 and 240 that is a multiple of 9.

Jake and Darren did a sponsored run. Jake earned £5 for every complete mile he ran. Darren earned £6 for every complete mile. They each raised the same amount of money, which was over £40 but under £80. How much money did each boy raise? How many miles did each boy run?

Consider the numbers 20 and 12. What is their lowest common multiple? What is their highest common factor?

Use factors, when appropriate, to calculate mentally, e.g.:

$$35 \times 12 = 35 \times 2 \times 6$$

Talk me through an easy way to do this multiplication/division mentally. Why is knowledge of factors important for this?

How do you go about identifying the factors of a number greater than 100?

What is the same/different about these sequences:

4.3, 4.6, 4.9, 5.2, ...

16.8, 17.1, 17.4, 17.7, ...

9.4, 9.1, 8.8, 8.5, ...

I've got a number sequence in my head. How many questions would you need to ask me to be sure you know my number sequence? What are the questions?

Find a number between 230 and 240 that has a factor of 9.

List all the factors of 36. How many does it have? Most numbers have an even number of factors. Why is 36 a special case?

I am thinking of a number that is a factor of 24 and a factor of 40. What is the largest possible number I could be thinking of?

Pupils to investigate patterns and spatial representations of known sequences of numbers, e.g. multiples, square numbers and triangular numbers.

Jon says, 'a square number cannot be an odd number.' Is Jon correct? How do you know?

What number squared gives me the answer of 81?

How could I calculate the square number for a large 3 digit number such as 123?

Give two prime numbers that add together to make ...?

Name a prime number greater than 100. How did you do it?

Ashad says 172 is a prime number. Is he right? How do you know?

How do you work out whether a large 3 digit number is a prime number?

### Number – Solving Numerical Problems

How many rolls of wallpaper would we need to cover the classroom walls?

Can you make a scale model of the length of the River Thames?

Work out the cost of a holiday to Spain for a week.

Costing a class party; the food, the tableware and the drink.

Deciding how much wood is needed to build a new shed for all the outdoor toys.



## Number- Fractions (including decimals and percentages)

When looking at graphs and handling data, ask questions such as what fraction of children like ... etc.

Ask pupils questions to show understanding such as 'is it better to have ... or ... of a chocolate bar or an amount of money. Explain why.

Jamie says  $\frac{2}{3}$  is smaller than  $\frac{20}{30}$  because the numbers are smaller. How can you help Jamie understand his mistake?

There are 360 people in a school. 20 of them are teachers. What fraction of the people are teachers? Write the fraction in its simplest form.

Show children visual patterns. Ask what fraction of the pattern is triangles, circles, rectangles etc. Write the fraction in its simplest form.

Ask pupils questions to show understanding such as:

When looking at graphs and handling data, ask questions such as what fraction of children like ... etc. Who likes... more? Which is the most popular...? Explain how you know.

Nazaria thinks that  $\frac{2}{30}$  is equivalent to  $\frac{1}{3}$  because the denominators are both in the three times table. Is she correct? Explain your answer and say how you would help Nazaria.

Using a large blank number line, ask children to generate their own fractions on post-it notes. They should place them in an appropriate place on the number line and justify their choice. They should explain why some fractions are larger or smaller and how they know.

Ask questions to do with money and discounts such as, 'In a sale, is it better to have  $\frac{2}{3}$  or  $\frac{3}{5}$  off of the cost of a laptop?' Explain your answer.

Ask pupils questions to show understanding such as:

Xavier has ... of £10 and Greg has ... of £10. How much do they have altogether?

Or

Grace has ... of pizza and Pia has .... How much have they eaten altogether?

There are 360 children in the school.  $\frac{2}{5}$  are boys. How many children are girls?  $\frac{4}{10}$  of the children enjoy football the most. How many children is this?

A roll of fabric is 4 metres long. A customer would like  $\frac{3}{10}$  cut off. How much material does she take home?

Draw me a bar chart of favourite crisps. 120 people were asked.  $\frac{1}{4}$  said salt and vinegar.  $\frac{1}{5}$  said cheese and onion.  $\frac{3}{10}$  said ready salted.  $\frac{1}{10}$  said prawn cocktail.  $\frac{3}{24}$  said bacon and the final  $\frac{1}{40}$  said none.

Would you prefer  $\frac{1}{4} \times 28$  or  $\frac{3}{8} \times 24$ ? Explain why.

Which is the better deal;  $\frac{7}{20}$  or  $\frac{9}{25}$  of a metre of ribbon for £1?

Would you prefer to share  $\frac{1}{2}$  pizza with 2 people or  $\frac{3}{4}$  of a pizza with 4 people? Explain your answer.

In a class of children half wanted to drink water and the other half juice. Of the half who wanted juice, a quarter wanted lemon and a third wanted orange. What fraction of the class wanted lemon and what fraction wanted orange?

If you divide a fraction by a whole number, does the answer get bigger or smaller? Explain why.

Would you prefer  $\frac{5}{8}$  of a bar of chocolate shared between 4 friends or  $\frac{3}{4}$  of a bar of chocolate shared between 3 friends?

Miss Tibbs is doing some baking with her class. She has seven groups of children. Each group will need one quarter of a block of butter. How many quarters does Miss Tibbs need altogether? Write as an improper and a mixed number fraction.

Milly is helping her mum paint an octagonal summer house. Each face requires  $\frac{1}{5}$  of a tin of paint. There are eight faces. How many fifths does Milly's mum need? How many tins of paint should she buy?

A small jar of jam will spread 6 slices of bread. A loaf has 20 slices of bread. How many slices of bread will 8 jars of jam spread? Write your answer as a mixed number and as an improper fraction.

Have a clothing sale with 10% off each item – what will they all cost?

Change a recipe to 50% of the original ingredients or add on 25% etc.

### Measurement

Possible contexts include practical experience for example:

using precise measurements for mass and capacity or volume;

measuring the growth of plants in a science experiment;

measuring jumps or throws in PE;

following a recipe where it is necessary to convert units;

converting a recipe for someone;

measuring distances in both miles and kilometres;

measuring and giving both values by converting;

recording temperatures in the classroom over a day to monitor changes

Give children problems such as:

How many cups holding 125ml can be filled from a 4.5l jug etc.

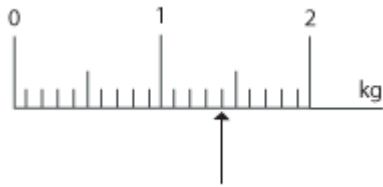
How many 30g blocks of chocolate will weigh 1.5kg?

Work out approximately how many kilometres are equivalent to 20 miles.

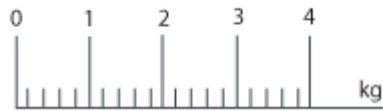
How many pint glasses can be filled from a 3 litre bottle?

A baby weighs 2.2kg. What is the baby's weight in pounds?

On this scale, the arrow shows the weight of a pineapple.



Here is a different scale. Mark with an arrow the weight of the same pineapple.



The carpet in Walt's living room is square, and has an area of  $4\text{m}^2$ . The carpet in his hall has the same perimeter as the living room carpet, but only 75% of the area. What are the dimensions of the hall carpet?

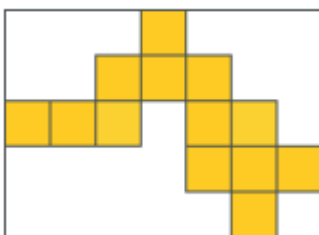
A square has a perimeter of 64 cm. How long is each side?

The perimeter of a rectangle is 72 cm. The shortest side is 9 cm. What is the length of the longest side?

An oblong has a perimeter of 24 cm. What length could the sides be?

What is the rectangle with the greatest area you can create if you have a perimeter of 24 cm?

If the length of each square is 8 cm, what is the area and what is the perimeter of the whole shape?



My rectangular garden is 11 m by 12 m. What is its area?

The base of a rectangular swimming pool has an area of  $275\text{ m}^2$ . It is 25 m long. How wide is it?

I want to rent an allotment and have two to choose from. One is 20 m by 15 m and the other is 10 m by 32 m. If I want the largest amount of space possible, which should I choose and why?

A square has an area of  $64 \text{ cm}^2$ . I cut it in half from one corner to the other to form two right angled triangles. Calculate the area of one triangle.

The area of a rectangle is  $72 \text{ cm}^2$ . I cut it into two equal right angled triangles. What is the area of one triangle?

A cube has an area of  $125 \text{ cm}^3$ . How long is each edge?

The volume of a cuboid is  $36 \text{ cm}^3$ . The longest edge is  $6 \text{ cm}$ . What are the lengths of the other two edges? Is there more than one possibility?

A cuboid has an area of  $24 \text{ m}^3$ . What length could the edges be?

My swimming pool is  $11 \text{ m}$  by  $12 \text{ m}$  with a depth of  $3 \text{ m}$ . What is its volume?

The base of a rectangular swimming pool has an area of  $275 \text{ m}^2$ . It is  $2.5 \text{ m}$  deep. What is its volume?

I need a large storage box. Box A has the dimensions  $125 \text{ cm}$ ,  $50 \text{ cm}$  and  $30 \text{ cm}$ . Box B has the dimensions  $100 \text{ cm}$ ,  $60 \text{ cm}$  and  $25 \text{ cm}$ . Which box will hold the most?

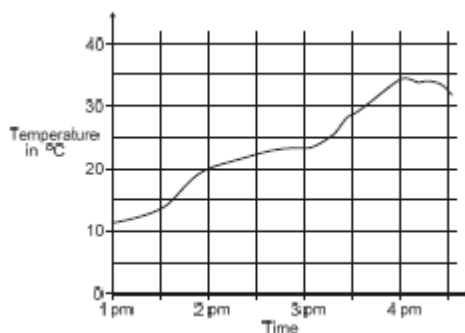
Ask questions that encourage children to work out time durations such as:

A film starts at  $6.45 \text{ pm}$ . It lasts  $2 \text{ hours}$  and  $35 \text{ minutes}$ . What time will the film finish?

Ask questions involving conversion between the  $24 \text{ hour}$  and  $12 \text{ hour}$  clock such as:

My train leaves at  $18.15$ . It is  $1.35 \text{ pm}$ . How long have I got until my train departs?

Encourage children to calculate times shown on graphs:

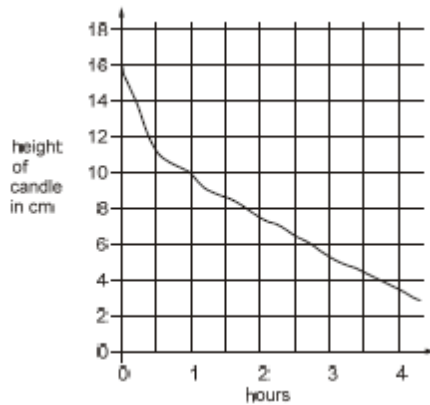


This graph shows the temperature in a greenhouse.

Use the graph to find the time when the temperature was  $25^{\circ}\text{C}$ .

Use the graph to find the difference between the temperature at  $2 \text{ pm}$  and the temperature at  $4 \text{ pm}$ .

This graph shows the height of a candle as it burns



What is the height of the candle after two hours?

How long does it take for the candle to burn down from 16cm to 4cm?

### Geometry- Properties of Shape

Opportunities to make boxes of different sizes and shapes to best fit objects and then explain why you chose that particular size and shape of box.

### Geometry- Position and Direction

Reflect the same shape in a series of parallel mirror lines. What is the same about the image each time? What is different?

If you reflect a line in a parallel mirror line, the image will always be parallel with the original line: true or false? Explain your reasoning

If you reflect a line in a vertical mirror line, the image will always be vertical: true or false? Explain your reasoning

Investigate the statement: 'All regular 2D polygons have the same number of lines of symmetry as they do sides.'

Translate a shape 2 right and 3 up. Then translate the image 5 left and 1 down. What single translation has the same effect as these two translations combined?

Translate a shape 3 left and 2 down. What translation must you now do to the image so that the combined effect of both translations is a single translation of 7 left and 4 up?

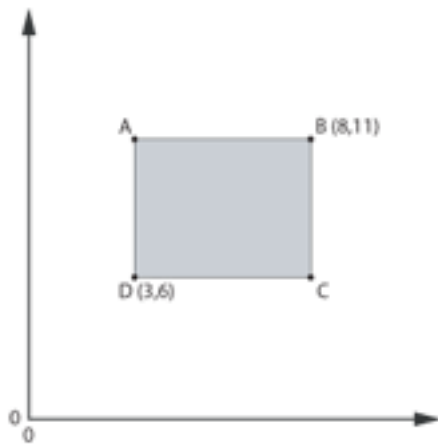
These points are the coordinates of the vertices of a shape:  $(1, 5)$ ,  $(2, 5)$ ,  $(4, 3)$ ,  $(2, 1)$ ,  $(1, 1)$ . What is the name of the shape?

If  $(6, 5)$  and  $(8, 5)$  are two vertices of a square, how many possibilities are there for the pair of missing vertices? What are they?

Three of the four vertices of a square are  $(3, 10)$ ,  $(5, 12)$  and  $(7, 10)$ . Work out the coordinates of the fourth vertex.

What could the coordinates be for a square with sides that are not parallel or perpendicular to the axes?

Here is a shaded square.



Write the coordinates for point A and point C. Explain how you worked them out.