## POLICY DOCUMENT

## Mathematics and Calculations

|  | Name | Date |
| :--- | :--- | :--- |
| Written By | SLT | Dec 2010 |
| Approved FGB / Sub- <br> Committee v1.0 | ACC | Jan 2011 |
| Review v1.1 | Liz Hughes | March 2014 |
| Review Version 1.2 | Bobbie Simpson, Chair of ACC | July 2015 |
| Review Version 1.3 | Bobbie Simpson, Chair of ACC | January 2016 |
| Review Version 1.4 | Phil le Conte, Carly Mould, Natalie Talbot, Jo <br> Wilcox | January 2020 |

This policy for Mathematics reflects the current philosophy and agreed practice of all staff at The Spring Partnership Trust member schools. It reflects the statutory requirements of the National Curriculum 2014 and takes into consideration the statutory framework for the Early Years Foundation Stage. The policy has the full agreement of the Governors and Directors and its implementation is the responsibility of all staff.

## Purpose of study (taken from the National Curriculum - September 2013)

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

## Aims

At The Spring Partnership Trust we:

- promote fluency and number sense
- promote confidence in mathematics by using a range of resources and methods
- develop deeper understanding through using the CPA approach (see appendix)
- use appropriate mathematical vocabulary
- encourage children to develop reasoning, pattern recognition and problem solving skills
- use mathematics in a range of contexts
- appreciate the importance of mathematics in everyday life
- apply mathematical knowledge across the curriculum


## Equal Opportunities

Schools in The Spring Partnership Trust want all children to enjoy mathematics and to regard themselves as mathematicians. We ensure that every child no matter what their gender, race, religion, culture or abilities has equal access to the mathematics curriculum.

## Organisation

The National Curriculum 2014 is at the centre of our mathematics teaching supported by a number of teaching resources. Daily mathematics lessons are taught in all Key Stage 1 and 2 classes. Year 1 follow an EYFS approach in the autumn term where appropriate (see below).

In The Early Years Foundation Stage we use the statutory framework for The Early Years Foundation Stage as a basis for our teaching. There is a focus on activities that are concrete, pictorial and abstract where mathematical understanding is developed through stories, songs, games and imaginative play. This approach continues in Year 1 in the autumn term. Manipulatives (such as Numicon, Dienes, cubes and place value counter) are used widely in the Foundation Stage to support children's understanding of number and calculation. The use of these manipulatives continues throughout KS1 and into KS2 to support the process of understanding and mastery.

## Planning

In order for the children to acquire key mathematical concepts and skills effectively, careful and comprehensive planning is essential. This planning will be completed for each year group, including Early Years Foundation Stage, and is on three connected levels:
a) Long term plans follow the White Rose Hub yearly overviews. These break down the National Curriculum into Units of teaching based on one key area of learning e.g. Fractions.
b) Medium term plans follow the White Rose Unit overviews. These outline the key objectives to be taught in each unit and the small steps required to achieve the objectives.
c) Short term planning, for each unit, are created by each year group utilising the planning ideas provided by White Rose, however teachers also access a number of other sources to inform and resource their planning. The short term plans are broken down into phases addressing linked key objectives within the unit.

Plans created are placed on the shared drive and made accessible to other schools within the trust to utilise and to be used as a starting point for subsequent years.

The key elements of effective planning include:

- Providing significant time to each unit - to enable each phase/objective to be taught in depth, demonstrating that concepts are broadened out, rather than accelerated to next year group objectives
- Having clear objectives and expected outcomes for the end of each unit and identifying the small steps of progression to fine-tune the teaching
- Lessons which are pitched appropriate to the year group and cross referenced against the trust's skills progression map
- Systematic outlining of teaching and learning through a process of Concrete Pictorial - Abstract
- Clearly identified Reasoning and Problem solving activities and questions which are brought into all lessons and are available to all children
- Outline of modelling and guided teaching, activities, including teacher role, challenge activities and any support given to groups, including manipulatives that might be used
- A range of contexts for learning in which learners are offered a variety of activities, resources and environments appropriate to their age, interests and prior achievements, making it purposeful and challenging
- Clear success criteria and assessment criteria that will enable children to assess their own progress and know how to improve
- Key vocabulary and language structures
- Key questions and opportunities for paired talk
- Outline of activities for lesson starters (where and when appropriate)
- Outline of main teaching input including modelling, resources and teaching techniques that will be used
- Brief details of how the class is to be organised, and where the attention of any support staff will be placed or split teaching might be employed
- Key points for the plenary and mini plenaries
- Evaluation and assessment by exception to inform future planning, particularly where learning objectives have not been achieved


## Teaching and Learning (see teaching and learning policy)

Our schools use a range of teaching and learning styles in mathematics to address the needs of all learners and ensure that they experience a broad and balanced mathematics curriculum. This is achieved through a daily mathematics lesson during which children are supported to:

- understand what they are learning, why they are learning it, how they will know when they have been successful and what they need to do to improve (see marking and feedback policy);
- ask as well as answer questions using appropriate mathematical vocabulary;
- use a process of CPA (Concrete-Pictorial-Abstract) to ensure full and deep understanding of mathematical concepts.
- experience a broad range of fluency, reasoning and problem solving questions where they are required to apply knowledge and skills in order to deepen their understanding of a topic;
- revisit topics in a structured manner to encourage retention of skills and knowledge;
- be given opportunities to apply more than one skill at any one time;
- make mathematical links with other curriculum areas and previous learning;


## Teaching mathematics to children with special educational needs

At our schools, we teach mathematics to all children whatever their ability by setting suitable learning challenges and responding to children's diverse needs. We provide learning opportunities that enable all children to make progress and overcome potential barriers to learning. The provision for children with special educational needs is detailed in the special educational needs and disability policy.

## Assessment

Rigorous assessment of children's learning in mathematics is a continuous process that is used to inform planning to ensure that children make at least good progress and reach their full potential.

Assessment for Learning is an integral part of every lesson to check children's understanding and to inform teachers, enabling them to adapt day-to-day lesson plans. Summative assessments take place at the end of each term in order to assess what children can do against national standards. These inform teachers' understanding of progress against national standards and help teachers to identify key marginal children. They are also used to inform planning and interventions.

Accurate information, including next steps for learning, will then be reported to parents termly and to the child's next teacher at the end of the academic year.

## The assessment procedures within our schools include:

- regular marking of children's work to provide feedback on the child's progress/ attainment (see Marking and Feedback Policy);
- use of PPR meetings to discuss children's progress and attainment;
- adjusting planning and teaching within units in response to pupils' attainment;
- use of the 'assessment for learning' strategies to check learning against objectives at the end of each unit of work. If necessary, future planning is adapted in response to assessment outcomes;


## Parents and Homework

Parents are informed termly of their child's progress, curricular targets and next steps at Parents' Evening. Parents are invited into school so their children can share their learning at their class assembly. We support parents through workshops and explain how they can help at home through meetings and newsletters. This policy is published on the school websites as well as links to useful maths websites.

Mathematics is part of the homework the children are given. Children are set tasks linked with the learning outcomes appropriate to their year group. Children are encouraged to use the homework to revise and consolidate learning done in class. From Year 2 upwards, children are also expected to frequently practice the multiplication tables appropriate to their year group.

## Roles and Responsibilities

- The monitoring of standards of children's learning and the quality of teaching in mathematics is carried out by the Subject Leader with the support of the Senior Leadership Team.
- Colleagues will be supported in the teaching of mathematics by the Subject Leader.
- The Subject Leader will keep up to date with new initiatives and ensure that teaching staff receive appropriate professional development.


## APPENDIX

## Maths Calculation Policy



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## About our Maths Calculation Policy

At Spring Partnership schools we strive to:

- Enable children to acquire mathematical skills and knowledge through a structured programme, ensuring progression through:
- promoting fluency in the understanding of numbers and number systems including mental strategies;
- promoting enjoyment, enthusiasm and confidence for learning through practical activity, exploration and discussion using appropriate mathematical vocabulary;
- developing a conceptual understanding of maths to enable them to apply it to problems in a range of contexts;
- encouraging children to reason effectively, making links between their learning developing problem solving in routine and non-routine situations and persevering to find a solution.
- developing a practical understanding of the ways in which information is gathered and presented;
- developing the skills to use patterns and identify relationships;
- exploring features of shape and space and develop measuring skills in a range of contexts;
- encouraging children to work both independently and cooperatively;
- appreciating the importance of mathematics in everyday life.
- To allow all children to fulfil their potential as mathematicians
- To apply mathematics knowledge to science and other subjects

The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school. Please note that early learning in number and calculations in EYFS follows the "Development Matters" EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.

## Age expectations

The calculation policy is organised according to age expectations (stages indicating year group expectations) as set out in the National Curriculum 2014, however it is vital that pupils are taught according to the stage that they are currently working at, being moved onto the next level as soon as they are ready or working at a lower stage until they are secure enough to move on.

## Choosing a calculation method

Children need to be taught and encouraged to use the following processes in deciding what approach they will take to a calculation, to ensure they select the most appropriate method for the numbers involved:

| Can I do it in my <br> head using a mental <br> strategy? <br> Could I use practical <br> resources help me? | Could I use some <br> jottings to help find <br> the answer? |
| :--- | :--- | :--- | :--- |

## Addition

## Stage 1: Add with numbers up to 20

Use numbered number lines to add, by counting on in ones. Encourage children to start with the larger number and count on.
$8+6=14$
Either:


Or:


## Children should:

Have access to a wide range of counting equipment, everyday objects, numicon and number lines, and be shown numbers in different contexts.

Read and write the addition (+) and equals (=) signs within number sentences.
Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them:
$7+5=$ $\qquad$ $12+4=$$6+4+2=\square$

$$
+\square=6
$$

This builds on from prior learning of adding by combining two sets of objects into one group (e.g. 3 cubes and 5 cubes) in EYFS.

When calculating $8+5$, for example, bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3 .


## Key Vocabulary:

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

## - Key Skills for addition at Stage 1:

- Read and write numbers to 100 in numerals, including 1-20 in words
- Recall bonds to 10 and 20, and addition facts within 20
- Count to and across 100
- Count in multiples of $1,2,5$ and 10
- Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations.


## Addition

## Stage 2: Add with 2-digit numbers

Children should develop mental fluency with addition and place value involving 2-digit numbers, moving on to establish more formal written methods.

## Add 2-digit numbers and tens:

$$
48+40=\mathbf{8 8}
$$



## Add 2-digit numbers and ones:

$18+6=24$


Use empty number lines, concrete equipment, number lines etc. to build confidence and fluency in mental addition skills.

Add pairs of 2-digit numbers, moving to the partitioned column method when secure:

Step 1: Only provide examples that do NOT cross the tens boundary until they are secure with the method itself.

$$
52+15=67
$$


$\mathbf{2 3}+\mathbf{3 4}=\mathbf{5 7}$

$$
\begin{array}{r}
20+3 \\
+30+4 \\
\hline 50+7=57
\end{array}
$$



To support understanding, pupils may physically make and carry out the calculation with Dienes Base 10 apparatus or numicon, then compare their practical version to the written form, to help them to build an understanding of it.


## Key Vocabulary:

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary

## - Key Skills for addition at Stage 2:

- Add a 2-digit number and ones (eg 27+6)
- Add a 2-digit number and tens (eg 23+40)
- Add pairs of 2-digit numbers (eg 35+47)
- Add three single-digit numbers (eg $5+9+7$ )
- Show that adding can be done in any order (the commutative law)
- Recall bonds to 20 and bonds of ten to 100 (eg 30+ 70 etc)
- Count in steps of 2, 3, 5 and count in tens from any number
- Understand the place value of 2-digit numbers (tens and ones)
- Compare and order numbers to 100 using $\leq, \geq$ and $=$ signs
- Read and write numbers to at least 100 in numerals and words
- Solve problems with addition, using concrete objects pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.
Addition


## Stage 3: Add numbers with up to 3-digits

Introduce the expanded column addition method:


Children need to recognise the value of the hundreds, tens and ones in order to record the partitioning.

Move to the compact column addition method, with "carrying":
Children who aresecure with 3-digit expanded column addition should be 374 moved onto the compact column addition method, being introduced to +32 "carrying" for the first time compare the expanded method to the compact $\frac{406}{1}$ number of steps involved.

```
Add the ones first
```

Remind children the actual value is "seven tens add three tens" which equals ten tens, not "three add seven" which equals one ten.

```
"Carry" numbers underneath the bottom
```



## Key Vocabulary:

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary increase, vertical, carry, expanded, compact

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## Addition

## Stage 4: Add numbers with up to 4-digits

Children should move from the expanded addition to the compact column method, adding ones first, and "carrying" numbers underneath the calculation. This method should be used in the context of money and measures.
eg $3517+396+3913$


Reinforce correct place value by reminding them the actual value is 3 hundreds add 2 hundreds, not 5 add 3.

## Key Vocabulary:

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse

## Key Skills for addition at Stage 4:

- Select the most appropriate method: mental, jottings or written and explain why
- Recognise the place vale of each digit in a 4-digit number
- Round any number to the nearest 10,100 , or 1000
- Estimate and use inverse operations to check answers
- Solve 2-step problem, in context deciding which operations and methods to use and why
- Find 1000 more or less than a given number
- Continue to practise a wide range of mental additional strategies i.e. number bonds add the nearest multiple of $10,100,1000$ and adjust use near doubles partitioning and recombining
- Add numbers with up to 4 digits using the formal written methods of column addition


## Addition

## Stage 5: Add numbers with more than 4-digits

Children should use column addition to solve problems including money, measures and decimals with varying numbers of decimal places.


Children should understand the place value of tenths and hundredths and use this to align numbers with different numbers of decimal places.

## Key Vocabulary:

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths hundredths, thousandths

## - 1 m <br> Key Skills for addition at Stage 5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies i.e. add the nearest multiple of $10,100,100$ and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds
- Use rounding to check answers and accuracy
- Solve multi-step problems in contexts deciding which operations and methods to use and why
- Read write, order and compare numbers to at least 1 million and determine the value of each digit
- Round any number up to 1000000 to the nearest $10,100,1000,10000$ and 100000
- Add numbers with more than 4 digits using formal written method of columnar addition


## Addition

## Stage 6: Add several numbers of increasing complexity



Adding several numbers with more than 4 digits

## Key Vocabulary:

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, addition, column, tens boundary, hundreds boundary increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse and decimal places, decimal point, tenths hundredths, thousandths

## Key Skills for addition at Stage 6:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies
- Solve multi-step problems in context, deciding which operations and methods to use and why
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Read, write, order and compare numbers up to 10 million and determine the value of each digit
- Round any whole number to a required degree of accuracy
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity


## Subtraction

## Stage 1: Subtract from numbers up to 20

Children will consolidate understanding of subtraction practically, showing subtraction using cubes, bead strings etc. and in familiar contexts, and are introduced to more formal recording using number lines as below:

## Subtract by taking away

Count back in ones on a numbered number line to
$9-5=4$


Model subtraction using numbered number lines and practically
This will be introduced practically with the language "find the difference between" and "how many more?" in a range of familiar contexts

"Six is 4 more than 2" 6 2
"I am 4 years older than my sister"

## Key Vocabulary:

Equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is $\qquad$

Key Skills for addition at Stage 1:

- Given a number, say one more or one less
- Count to and over 100 , forward and back, from any number
- Represent and use subtraction facts to 20 and within 20
- Subtract with 1-digit and 2-digit numbers to 20 , including zero
- Solve 1-step problems that involve addition and subtraction, using concrete objects (i.e. numicon, bead string, objects, cubes) and pictures, and missing number problems
- Read and write numbers from 0 to 20 in numerals and words


## Subtraction

## Stage 2: Subtract with 2-digit numbers

Children will subtract on a number line by counting back, aiming to develop mental subtraction skills
This strategy will be used for:

- 2-digit numbers subtract ones (by taking away / counting back) eg 36-7
- 2-digit numbers subtract tens (by taking away / counting back) eg 48-30
- Subtracting pairs of 2-digit numbers (see below)


## Subtracting pairs of 2-digit numbers on a number line:

$78-45=33$ Partition the second number and subtract it in tens and ones, as below:


Move towards more efficient jumps back, as below:


Combine methods with use of a hundred square to reinforce
Teaching children to bridge thro understanding of number value and order.
$78-45=33$


Mental strategy - subtract numbers close together by counting on:
$52-48=4$

## Start here



Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to count on the difference. They need to be clear about the relationship between addition and subtraction.

Start with the smaller number and coun on to the largest.

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## Key Vocabulary:

Equal to, take, take away, less, minus, subtract, leaves, how many more, how many fewer / less than, most, least, count back, how many left, how much less is $\qquad$ ?, difference, count on, strategy, partition, tens, ones

## - Key Skills for subtraction at Stage 2:

- Recognise the place value of each digit in a 2-digit number
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100
- Subtract using concrete objects, pictorial representations, number lines and mentally including: a 2-digit number and ones, a 2-digit number and tens, and two 2-digit numbers
- Show that subtraction of one number from another cannot be done in any order
- Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods
- Read and write numbers to at least 100 in numerals and in words


## Subtraction

## Stage 3: Subtracting with 2 and 3-digit numbers

Introduce partitioned column subtraction method.
Step 1: introduce this method with examples where no exchanging is required.

Step 2: introduce "exchanging" through practical subtraction. Make the larger number with Base 10/numicon, then subtract 47 from it.


Step 3: Once pupils are secure with the understanding of "exchanging", they can use the partitioned column method to subtract any 2 and 3-digit.

457-162 = 295


## Counting on as a mental strategy for subtraction:

Continue to reinforce counting on as a strategy for close together numbers (e.g. 132-127), and also for numbers that are "nearly" multiples of $10,100,1000$ or $£ s$, which make it easier to count on (e.g. 202191, 143-89, or calculating change from $£ 1$ etc.) Start at the smaller number and count on in tens first, then count on in ones to find the rest of the difference:


## Key Vocabulary:

Equal to, take, take away, less, minus, subtract, leaves, how many more, how many fewer / less than, most, least, count back, how many left, how much less is __?, difference between, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit


## Key Skills for subtraction at Stage 3:

- Subtract mentally a 3-digit number and ones, 3-digit number and tens, a 3-digit number and hundreds
- Estimate answers and use inverse operations to check
- Solve problems, including missing number problems
- Find 10 or more or less than a given umber
- Recognise the place value of each digit in a 3-digit number
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above)
- Read and write numbers up to 1000 in numerals and words
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21 ) and select most appropriate methods to subtract, explaining why.


## Subtraction

Partitioned column subtraction with "exchanging" (Decomposition)


## Compact column subtraction



To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different, to develop an understanding of it.

Always encourage children to consider the best method for the numbers involved - mental, counting on, counting back or written method.

## Mental strategies

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are close together. Children should approximate first.

## Key Vocabulary:

Equal to, take, take away, less, minus, subtract, leaves, how many more, how many fewer / less than, most, least, count back, how many left, how much less is __?, difference, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit, inverse

## Key Skills for subtraction at Stage 4:

- Subtract by counting on where numbers are close together or they are near to multiples of 10,100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations
- Estimate and use inverse operations to check answers
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why
- Solve simple measure and money problems involving fractions and decimal to two decimal places
- Find 1000 more or less than a given number
- Count backwards through zero, including negative numbers
- Recognise place value of each digit in a 4-digit number. Round any number to the nearest 10,100 or 1000
- Solve number and practical problems that involve the above, with increasingly large positive numbers


## Subtraction

## Stage 5: Subtract with at least 4-digit numbers

Including money, measures and decimals
Compact column subtraction (with "exchanging")


## Subtracting with large numbers



Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

## - Key Vocabulary:

Equal to, take, take away, less, minus, subtract, leaves, how many more, how many fewer / less than, most, least, count back, how many left, how much less is __?, difference between, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

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## Key Skills for subtraction at Stage 5:

- Subtract numbers mentally with increasingly large numbers
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit
- Count forwards or backwards in steps of powers of 10 for any given umber up to 1 million
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0
- Round any number up to 1 million to the nearest $10,100,1000,10000$ and 100000


## Subtraction

## Stage 6: Subtracting with increasingly large and more complex numbers and decimal values



Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting the most appropriate method to work out subtraction problems.

## - Key Vocabulary:

Equal to, take, take away, less, minus, subtract, leaves, how many more, how many fewer / less than, most, least, count back, how many left, how much less is __?, difference, count on, strategy, partition, tens, ones, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

## Key Skills for subtraction at Stage 6:

- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why
- Read, write, order and compare numbers up to 10 million and determine the value of each digit
- Round any whole number to a required degree of accuracy
- Use negative numbers in context, and calculate intervals across zero
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate


## Multiplication

## Stage 1:Multiply with concrete objects, arrays and pictorial representations

How many legs will 3 chickens have?


There are 3 sweets in one bag. How many sweets are in 5 bags altogether?


- Give children experience of counting equal groups of objects in $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s
- Present practical problem solving activities involving counting equal sets or groups, as above


## Key Vocabulary:

Groups of, lots of, times, array, altogether, multiply, count

Key Skills for multiplication at Stage 1:

- Count in multiples of 2,5 and 10
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- Make connections between arrays, number patterns and counting in $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s
- Begin to understand doubling using concrete objects and pictorial representations


## Multiplication

## Stage 2: Multiply using arrays and repeated addition (using at least 2s, 5 s and 10s)

## Use repeated addition on a number line:

Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and $=$ signs
$3 \times 4=3$ lots of 4

$3 \times 4=12$


Use arrays to help teach children to understand the commutative law of multiplication and give examples such as 3 x $\qquad$ $=6$

Use practical apparatus ${ }^{5} 00-0^{5} 00-0^{5} 00-0^{5} 00$

## Use mental recall:

Children should begin to recall multiplication facts for 2,5 and $\mathbf{1 0}$ times tables through practice in counting and understanding of the operation.

## Key Vocabulary:

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition column, row, commutative, sets of, equal groups, times as big as, once, twice, three times

## - Key Skills for multiplication at Stage 2:

- Count in steps of 2,3 and 5 from zero, and in 10 s from any number
- Recall and use multiplication facts from the 2,5 and 10 multiplication tables, including recognising odds and evens
- Write and calculate number statements using the x and = signs
- Show that multiplication can be done in any order (commutative)
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods and multiplication facts
- Pupils use a variety of language to discuss and describe multiplication


## Multiplication

## Stage 3: Multiply 2-digits by a single digit number:

Introduce the grid method for multiplying 2-digit by single digits:
$\mathbf{3 2} \times 7=224$


Link the layout of the grid to an array initially:


Introduce the grid method with children physically making an array to represent the calculation (e.g. make 6 lots of 13 with 10 s and 1 s place value counters), then translate this to grid method format.

## To do this, children must be able to:

- Partition numbers into tens and ones
- Multiply multiples of ten by a single digit (e.g. $30 \times 4$ ) using their knowledge of multiplication facts and place value
- Recall and work out multiplication facts in the $\mathbf{2 , 3 , 4 , 5 , 8}$ and 10 times tables
- Work out multiplication facts now known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.)

Strategies to support this are repeated addition using a number line, bead bards and arrays:


## Key Vocabulary:

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, multiple, product, tens ones, value

```
Key Skills for multiplication at Stage 3:
- Recall and use multiplication facts for the \(2,3,4,5,8\) and 10 multiplication tables, and multiply multiples of 10
- Write and calculate number statements using the multiplication table s they know, including 2digit x single-digit, drawing upon mental methods, and progressing to reliable written methods
- Solve multiplication strategies using commutativity (e.g. \(4 \times 12 \times 5=4 \times 5 \times 12=20 \times 12=\) 240)
- Solve simple problems in contexts, deciding which operations and methods to use
- Develop efficient mental methods to solve a range of problems e.g. using commutativity ( \(4 \times 12\) \(\mathrm{x} 5=4 \times 5 \times 12=20 \times 12=240\) ) and for missing number problems \(\quad\) x \(5=20,3 \mathrm{x} \quad=18\), \(\chi^{\mathrm{x}} \mathrm{C}^{=} 32\)
```


## Multiplication

## Stage 4: Multiply 2 and 3-digit numbers by a single digit, using multiplication tables up to $12 \times 12$ :

Developing the grid method:
$145 \times 6=870$


## Encourage column addition to add accurately

Move onto short multiplication (see Year 5) if and when children are confident and accurate multiplying 2 and 3 digit numbers by a single digit this way, and are already confident in "carrying" for written addition.

## Children should be able to:

- Approximate before they calculate, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer e.g.
$\mathbf{2 4 6} \mathbf{x} \mathbf{8}$ is approximately $250 \times 10=\mathbf{2 5 0 0}$
- Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge
- Recall all times tables up to $12 \times 12$

Key Vocabulary:
Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, total, multiple, product, sets of, inverse

## - Key Skills for multiplication at Stage 4:

- Count in multiples of $6,7,9,25$ and 1000
- Recall multiplication facts for all multiplication tables up to $12 \times 12$
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by $1,10,100$, by 0 , or to multiply 3 numbers
- Use commutativity and other strategies mentally $3 \times 6=6 \times 3,2 \times 6 \times 5=10 \times 6$, $39 \times 7=30 \times 7+9+7$
- Solve problems with increasingly complex multiplication in a range of contexts
- Recognise the place value of each digit in a 4-digit number (thousands, hundreds, tens, and ones)


## Multiplication

## Stage 5: Multiply up to 4 -digits by 1 or 2 digits:

## Introducing column multiplication

- Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are less steps involved in the column method
- Children need to be taught to approximate first, e.g. for $72 \times 38$, they will use rounding: $72 \times 38$ is approximately $70 \times 40=2800$, and use the approximation to check the reasonableness of their answer against


## Short multiplication for multiplying by a single digit

$473 \times 4$ - $=1892$


Pupils could be asked to work out a given calculation using the grid and then compare it to "your" column method What are the similarities and differences? Unpick the steps and show how it reduces the steps.

## Introduce long multiplication for multiplying by 2 digits

$17 \times 15=255$

| $\times$ | 10 | 7 | 17 | $17 \times 5$ on the $1^{\text {st }}$ row <br> (7 $\times 5=35$, carrying the 3 for thirty, then $1 \times 5$ ) |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 100 | 70 | $\frac{185}{170}$ | $10 \times 17$ on the $2^{\text {nd }}$ row. Put a zero in ones first, then say $7 \times 1$, and then $1 \times 1$. |

## Moving towards more complex numbers



## Key Vocabulary:

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, total, multiple, product, sets of, inverse, square, factor, integer, decimal, short/long multiplication, "carry"

Key Skills for multiplication at Stage 5:

- Identify multiples and factors, using knowledge of multiplication tables to $\mathbf{1 2} \times 12$
- Sole problems where larger numbers are decomposed into their factors
- Multiply and divide integers and decimals by 10, 100 and 1000
- Recognise and use square and cube numbers and their notation
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately


## Multiplication

## Stage 6: Short and long multiplication as in Stage 5, and multiply decimals with up to 2 decimal places by a single digit:



- Line up the decimal points in the question and the answer
- This works well for multiplying money (£.p) another other measures
- Remind children that single digit belongs in the ones column


## Children will be able to:

- Use rounding and place value to make approximations before calculating and use these to check answers.
- Use short multiplication (see Year 5) to multiply numbers with more than 4-digits by a single digit; to multiply money and measures, and to multiply decimals with up to 2 decimal places by a single digit.
- Use long multiplication (see Year 5) to multiply numbers with at least $\mathbf{4}$ digits ba 2-digit number.


## Key Vocabulary:

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, total, multiple, product, sets of, inverse, square, factor, integer, decimal, short/long multiplication, "carry", tenths, hundredths, decimal

Key Skills for multiplication at Stage 6:

- Recall multiplication facts for all times tables up to $12 \times 12$ (as stage 4 and stage 5 )
- Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication
- Perform mental calculations with mixed operations and large numbers
- Sole multi-step problems in a range of contexts choosing appropriate combinations of operations and methods
- Estimate answers using round and approximations and determine levels of accuracy
- Round any integer to a required degree of accuracy


## Division

## Stage 1: Group and share small quantities:

Using objects, diagrams and pictorial representations to solve problems involving both grouping and sharing.

## Grouping:

How many groups of 4 can be made with 12 stars? 3


Sharing:
9 shared between 3 is 3


## Example division problem in a familiar context:

There are 6 pupils on this table and there are 18 pencils to share between us. If we share them equally, how many will we each get?

Can they work it out and give a division statement?
" 18 shared between 6 people gives you 3 each".

## Pupils should:

- Use lots of practical apparatus, arrays and picture representations
- Be taught to understand the difference between "grouping" objects (How many groups can you make?) and "sharing" (Share these sweets between 2 people)
- Be able to count in multiples of $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s
- Find half of a group of objects by sharing into 2 equal groups



## Key Vocabulary:

Share, share equally, one each, two each, group, groups of, lots of, array


## Key Skills for division at Stage 1:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, pupils begin to understand division and finding simple fractions of objects, numbers and quantities
- They make connections between arrays number patterns, and counting in $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s


## Division

## Stage 2: Group and share, using the $\div$ and $=$ sign:

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line

Arrays:


This represents $12 \div 3$, posed as how many groups of 3 are in 12 ?
Pupils should also show that the same array can represent $12 \div 4=3$ if grouped horizontally

## Know and understand sharing and grouping:

6 sweets shared between 2 people, how many do they each get?

## Sharing



Children should be taught to recognise whether problems require sharing or grouping.

## Grouping using a number line:



Group from zero in equal jumps of the divisor to find out "how many groups of _in_?"
Pupils could using a bead string or practical apparatus work out problems like "A magazine costs $£ 3$. How many magazines can I buy with £12?"

This is an important method to develop understanding of division as grouping.

Pose $12 \div 3$ as "How many groups of 3 are in 12 ?"


## Key Vocabulary:

Share, share equally, one each, two each, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

## 

## Key Skills for division at Stage 2:

- Count in steps of 2, 3 and 5 from 0
- Recall and use multiplication and division facts for the $\mathbf{2 , 5}$ and $\mathbf{1 0}$ multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the $\mathrm{x}, \div$ and $=$ signs
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- Solve problems involving multiplication and division, using materials, arrays repeated addition, mental methods, and multiplication and division facts, including problems in contexts
Division


## Stage 3: Divide 2-digit numbers by a single digit (where initially there is no remainder in the final answer):

## Grouping on a number line:



Step 1: Children continue to work out unknown division facts by grouping on a number line from zero.

They are also now taught the concept of remainders, as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the $2 \mathrm{~s}, 3 \mathrm{~s}, 4 \mathrm{~s}, 5 \mathrm{~s}, 8 \mathrm{~s}$ and 10 s ready for "carrying" remainders across within the short division method.

## Short division:

Limit numbers to NO remainders in the answer OR carried (each digit must be a multiple of the divisor)
 division pose:

Step 2: Once children are secure with division as grouping and demonstrate this using number lines, arrays, etc., short division for large 2-digit numbers should be introduced, initially with carefully selected examples requiring no
Remind children of correct place value, that 69 is equal to $60+9$, but in short

- How many 3's in $6 ?=2$, record it above the 6 tens
- How many 3's in $9 ?=3$, and record it above the 9 ones


## Short division:

Limit numbers to $\mathbf{N O}$ remainders in the final answer, but with remainders occurring within the number being divided into


Step 3: Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. $96 \div 4$ ), and be taught to "carry" the remainder onto the next digit. If needed, children should use the number line to work out individual division facts that occur which they are not yet able to recall mentally.

Step 3: Only taught when pupils can calculate remainders

Real life contexts and practical activities need to be integral to the learning to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

## Key Vocabulary:

Share, share equally, one each, two each, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number lie, left, left over, inverse, short division, "carry", remainder, multiple

## - m

## Key Skills for division at Stage 3:

- Recall and use multiplication and division facts for the $2,3,4,5,8$ and 10 multiplication tables (through doubling, connect the 2, 4 and 8 s )
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know including for 2-digit numbers times 1-digit numbers, using mental and progressing to formal written methods
- Solve problems, in contexts and including missing number problems, involving multiplication and division
- Pupils develop efficient mental methods, for example using multiplication and division facts (e.g. $3 \times 2=6,6 \div 3=2$ and $2=6 \div 3$ ) to derive related facts ( $30 \times 2=$ 60 ,
so $60 \div 3=20$ and $20=60 \div 3$ )
- Pupils develop reliable written methods for division, starting with calculations of 2digit numbers by 1 -digit numbers progressing to the formal written method of short division


## Division

## Stage 4: Divide up to 3-digit numbers by a single digit (without a remainder initially):

## Continue to develop short division:

Short division should only be taught once children have secured the skill of calculating "remainders".


Step 1: Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit (those that do not result in a final remainder - see steps in Stage 3), but must understand how to calculate remainders, using this to "carry" remainders within the calculation process (see example)


When the answer for the first column is zero ( $2 \div 4$, as in example), children could initially write a zero above to acknowledge its place, and must always "carry" the number (2) over to the net digit as a remainder.

## Key Vocabulary:

Share, share equally, one each, two each, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number lie, left, left over, inverse, short division, "carry", remainder, multiple, divisible by, factor

## Key Skills for division at Stage 4:

- Recall multiplication and division facts for all numbers up to $12 \times 12$
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a 1 -digit number
- Pupils practise mental methods and extend this to 3-digit numbers to derive facts
- Pupils solve 2-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children


## Division

## Stage 5: Divide up to 4-digit numbers by a single digit (including those with remainders):

Short division, including remainder answers:


## Short division with remainders:

Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where pupils consider the meaning of the remainder and how to express it, i.e. as a fraction, a decimal, or
 as a rounded number or value, depending upon the context of the problem.

The answer to $3832 \div 7$ could be expressed as 547 and three sevenths,
547 r3, as a decimal, or rounded as appropriate to the problem involved.

Remainders should be appropriate to the context of the question.

## Key Vocabulary:

Share, share equally, one each, two each, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, "carry", remainder, multiple, divisible by, factor, quotient, prime number, prime factors, composite number (non-prime)

## - 1 m <br> Key Skills for division at Stage 5:

- Recall multiplication and division facts for all numbers up to $12 \times 12$ (as in Year 4)
- Multiply and divide numbers mentally, drawing upon known facts
- Identify multiple and factors, including finding all factor pairs of a number and common factors of two numbers
- Solve problems involving multiplication and division where larger numbers are recomposed into their factors
- Multiply and divide whole numbers and those involving decimals by 10,100 and 1000
- Use the vocabulary of prime numbers prime factors and composite (non-prime) numbers
- Work out whether a number up to 100 is prime, and recall prime numbers to 19
- Divide numbers up to 4 -digits by a 1 -digit number using the formal written method of short division and interpret remainders appropriately for the context
- Use multiplication and division as inverses
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. $98 \div 4=24 \mathrm{r} 2=241 / 2=24.5 \approx 25$ )
- Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates


## Division

## Stage 6: Divide at least 4-digit numbers by both single digit and 2-digit numbers (including decimal numbers and quantities):

Short division, for dividing by a single digit:

e.g. $7459 \div 4$



## Short division with remainders:

Pupils should continue to use this method but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Calculating a decimal remainder: In this example, rather than expressing the remainder as $\underline{r} 3$, a decimal point is added after the ones because there is still a remainder, and the one remainder is carried into zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.

Introduce short division by dividing a number by 2-digits:

$$
\begin{array}{r}
037 \\
1 4 \longdiv { 5 ^ { 5 } 1 ^ { 9 } 8 }
\end{array}
$$

When the answer for the first column is zero ( $5 \div 14$ ), as in example, children could initially write a zero above to acknowledge its place and must always "carry" the number (5) over to the net digit as a remainder.

When remainders occur, pupils should express them as fractions, decimals or use rounding, depending upon the problem.

## Introduce long division by chunking by 2-digits:



Find out "How many 15 s are in 432 ?" by subtracting "chunks" of 15 , until zero is reached (or until there is a remainder)

Teach pupils to write a "useful list" first at the side that will help them decide what chunks to use egg. Useful list: Ix $15=$ 15
$10 \times 15=150$
$20 \times 15=300$

Introduce the method in a simple way by limiting the choice of chunks to "Can we use 10 lots? Can we use 20 lots?" As children become confident with the process, encourage more efficient chunks to get to the answer more quickly (e.g. 100x, 30x), and expand on their "useful" lists.


Introduce long division by dividing any number by 2 digits:

15 does not fit in to 4 so we need to look at the next digit in 432.

15 goes in to 43 twice so put a 2 above the 3
$2 \times 15=30$
Subtract that 30 from the 43 to get your remainder.

## Key Vocabulary:

Share, share equally, one each, two each, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, "carry", remainder, multiple, divisible by, factor, quotient, prime number, prime factors, composite number (non-prime), common factor

## - Key Skills for division at Stage 6:

- Recall and use multiplication and division facts for all numbers to $12 \times 12$ for more complex calculations
- Divide numbers up to 4-digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate
- Perform mental calculations, including with mixed operations and large numbers
- Identify common factors, common multiples and prime numbers
- Solve problems involving all 4 operations
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem
- Use written division methods in cases where the answer has up to two decimal places
- Solve problems which require answers to be rounded to specified degrees of accuracy


[^0]:    Key Skills for addition at Stage 3:

    - Read and write numbers to 1000 in numerals and words
    - Add 2-digit numbers mentally, including those exceeding 100
    - Add a 3-digit number and ones mentally (eg 134+7)
    - Add a 3-digit number and tens mentally (eg 136+40)
    - Add a 3-digit number and hundreds mentally $(224+400)$
    - Estimate answers to calculations, using inverse to check answers
    - Solve problems, including missing number problems using number facts, place value, and more complex addition.
    - Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones)
    - Continue to practise a wide range of mental addition strategies, ie number bonds, adding to the nearest multiple of $10,100,100$ and adjusting, using near doubles, partitioning and recombining.

